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Vector Meson Photoproduction at High-t and Comparison to HERA Data

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We explore QCD calculations for the process $\gamma p \to VX$ where V is a vector meson, in the region $s \gg -t$ and $-t \gg \Lambda^2_{QCD}$. We compare our calculations for the J/ψ, ϕ and ρ mesons with preliminary data from the ZEUS Collaboration at HERA and demonstrate that the BFKL approach is consistent with the data even for light mesons, whereas the two-gluon exchange approach is inadequate. We also predict the differential cross-sections for the Υ and ω for which no data are currently available.

1 Introduction

We study the process $\gamma p \to V + X$ (where V is a vector meson: $V = \rho$, ω, ϕ, J/ψ or Υ) in the perturbative Regge limit, $s \gg -t \gg \Lambda^2_{QCD}$. The largeness of $-t$ allows the application of the solutions of the non-forward BFKL equation. We apply the analytic solutions to the cross-section arrived at in [1, 2] in the case of a delta-function distribution for the meson wavefunction. Specifically we apply this solution to the case of a real photon which has been measured at HERA [3, 4].

The cross-section for the helicity-flip process $\gamma p \to V_L X$ vanishes for a delta-function wavefunction and throughout this paper the cross-sections pertain to the process $\gamma p \to V_T X$ (where L and T correspond to longitudinal and transverse polarization respectively). This approximation is justifiable since the measured rate for longitudinal mesons is indeed small [3]. We make predictions and compare to experiment for the mesons for which data are available, the heavy J/ψ and the light ρ and ϕ (for which we might expect the delta-function to be a poor approximation). We show that the data for all the mesons can be well described by the BFKL approach. We then demonstrate that the two-gluon exchange model is incapable of adequately describing the data. Finally we make predictions for the Υ and ω.

The high momentum exchange allows us to factorize the cross-section into the usual product of the parton distribution functions and the parton level cross-section:

$$\frac{d\sigma(\gamma p \to VX)}{dt dx} = \left(\sum_f [q_f(x,t) + \bar{q}_f(x,t)]\right) \frac{d\sigma(\gamma q \to Vq)}{dt} + G(x,t) \frac{d\sigma(\gamma g \to Vg)}{dt}, \quad (1)$$

where $G(x,t)$ and $q_f(x,t)$ are the gluon and quark parton distribution functions respectively and we sum over flavour, f. For a large separation in rapidity between the parton and the vector meson we may write,

$$\frac{d\sigma(\gamma p \to VX)}{dt dx} = \left(\frac{4N_c^4}{(N_c^2-1)^2} G(x,t) + \sum_f [q_f(x,t) + \bar{q}_f(x,t)]\right) \frac{d\sigma(\gamma q \to Vq)}{dt}, \quad (2)$$

where we define the parton level amplitude by

$$\frac{d\sigma(\gamma q \to Vq)}{dt} \equiv \frac{\pi}{4t^4} |\mathcal{F}(s,t)|^2. \quad (3)$$

2 The Two Gluon and BFKL Amplitudes

We consider two types of colour singlet exchange: two-gluon and BFKL. The two-gluon amplitude can be expressed to leading order in s in the impact factor representation:

$$\mathcal{F}_{Born}(s,t) = 2\pi t^2 \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{\mathcal{I}_{\gamma V}\, \mathcal{I}_{qq}}{k_\perp^2 (k_\perp - Q_\perp)^2}, \quad (4)$$

where $\mathcal{I}_{\gamma V}$ and $\mathcal{I}_{qq}$ are the impact factors associated with the couplings of the two gluons to the external particles. Vectors with the subscript $\perp$ are two dimensional transverse momenta and $Q_\perp^2 = -t$. With the above definitions the impact factor for $q \to q$ is given by

$$\mathcal{I}_{qq} = \frac{\delta_{ab}}{N_c} \alpha_s \quad (5)$$

The $\gamma \to V_T$ impact factor, assuming a delta-function form for the meson wavefunction, is

$$\mathcal{I}_{\gamma V} = \mathcal{C}\, \alpha_s \frac{\delta_{ab}}{2N_c} \left(\frac{4}{Q_\perp^2 + M_V^2} - \frac{1}{(k_\perp - Q_\perp/2)^2 + M_V^2/4} \right), \quad (6)$$

where,

$$\mathcal{C}^2 = 3\Gamma^V_{e^+e^-} M_V^3/\alpha_{em}. \tag{7}$$

M_V is the mass of the vector meson and Γ^V_{ee} is the electronic decay width of the meson. Using these impact factors we obtain the two-gluon amplitude as a function of one dimensionless parameter, $\tau = -t/M_V^2$:

$$\mathcal{F}_{Born} = \mathcal{C}\,\mathcal{I}_{qq}^2\left(\frac{4\tau^2}{1-\tau^2}\right)\ln\left(\frac{(1+\tau)^2}{4\tau}\right). \tag{8}$$

The BFKL amplitude, in the leading logarithm approximation (LLA), is given by [5],

$$\mathcal{F}_{BFKL}(s,t) = \frac{t^2}{(2\pi)^3}\int d\nu \frac{\nu^2}{(\nu^2+1/4)^2} e^{\chi(\nu)z} I_\nu^{q^*}(Q_\perp) I_\nu^{\gamma V}(Q_\perp), \tag{9}$$

where,

$$\chi(\nu) = 4\mathcal{R}e\left(\psi(1) - \psi\left(\frac{1}{2}+i\nu\right)\right) \tag{10}$$

and

$$z = \frac{3\alpha_s}{2\pi}\ln\left(\frac{s}{\Lambda^2}\right). \tag{11}$$

In LLA, Λ is arbitrary (it need only be small compared to $\sqrt{s}$) and α_s is a constant. The impact factors are used, in conjunction with the prescription of [6], to give

$$\begin{aligned} I_\nu^A(Q_\perp) &= \int \frac{d^2k_\perp}{(2\pi)^2}\mathcal{I}_A(k_\perp,Q_\perp)\int d^2\rho_1 d^2\rho_2 \\ &\times \left[\left(\frac{(\rho_1-\rho_2)^2}{\rho_1^2\rho_2^2}\right)^{1/2+i\nu} - \left(\frac{1}{\rho_1^2}\right)^{1/2+i\nu} - \left(\frac{1}{\rho_2^2}\right)^{1/2+i\nu}\right] e^{ik_\perp\cdot\rho_1 + i(Q_\perp-k_\perp)\cdot\rho_2}, \end{aligned} \tag{12}$$

In the case of coupling to a colourless state only the first term in the square bracket survives since $\mathcal{I}_A(k_\perp, Q_\perp = k_\perp) = \mathcal{I}_A(k_\perp = 0, Q_\perp) = 0$ in this case. After some work, one obtains [2]:

$$I_\nu^q(Q_\perp) = -4\pi\mathcal{I}_{qq}\,2^{-2i\nu}|Q_\perp|^{-1+2i\nu}\frac{\Gamma(\frac{1}{2}-i\nu)}{\Gamma(\frac{1}{2}+i\nu)} \tag{13}$$

$$\begin{aligned} I_q^V(Q_\perp) &= -\mathcal{C}\,\mathcal{I}_{qq}\frac{\pi^2}{Q_\perp^3}\frac{\Gamma(1/2-i\nu)}{\Gamma(1/2+i\nu)}\left(\frac{Q_\perp^2}{4}\right)^{i\nu}\int_{1/2-i\infty}^{1/2+i\infty}\frac{ds}{2\pi i}\left(\frac{Q_\perp}{M_V/2}\right)^{1+2s} 2^{2(1-s)} \\ & \frac{\Gamma(1-s-i\nu)\Gamma(1-s+i\nu)\Gamma^2(1/2+s)}{\Gamma(1/2+s/2-i\nu/2)\Gamma(1-s/2-i\nu/2)\Gamma(1-s/2+i\nu/2)\Gamma(1/2+s/2+i\nu/2)}. \end{aligned} \tag{14}$$

Fit	α_s	β	$\chi^2_{J/\psi}$/dof	χ^2_ϕ/dof	χ^2_ρ/dof	χ^2/dof
A	0.25	13.5	0.25	0.4	0.5	0.4
B	0.17	0.081	2.6	0.5	0.6	1.0
C	0.29	70.6	1.0	0.4	1.1	0.9

Table 1: BFKL fits to the preliminary ZEUS data.

Putting these into (9) we obtain,

$$\mathcal{F}_{BFKL}(s,t) = 4\mathcal{C}\,\mathcal{I}_{qq}^2 \int d\nu \frac{\nu^2}{(\nu^2+1/4)^2} e^{\chi(\nu) z} \int_{1/2-i\infty}^{1/2+i\infty} \frac{ds}{2\pi i} \tau^{1/2+s} \qquad (15)$$
$$\frac{\Gamma^2(1/2+s)\Gamma(1-s-i\nu)\Gamma(1-s+i\nu)}{\Gamma(1/2+s/2-i\nu/2)\Gamma(1-s/2-i\nu/2)\Gamma(1-s/2+i\nu/2)\Gamma(1/2+s/2+i\nu/2)}.$$

3 Results and Discussion

Using the full numerical calculation for the amplitude we convoluted the partonic cross-section for the mesons with the parton density functions of the proton[1], integrating over x in the region $0.01 < x < 1$ (for which the ZEUS data are quoted). We obtained fits in terms of three free parameters. One parameter is α_s and the others appear in the denominator of the logarithm defining the energy variable z, i.e. we take $\Lambda^2 = \beta M_V^2 + \gamma|t|$.

Initially we tried the simple prescription of [1, 2] where $\beta = \gamma = 1$. The J/ψ has been fitted with $\alpha_s = 0.20$ for this parameterization previously [3][2] and as expected this gave a χ^2/dof < 1, however these parameters gave very poor fits to the ρ and ϕ. When we varied all three parameters independently we found that optimum fits took small values of γ and in fact we were able to put this parameter to zero. The optimum fit for this two parameter model is given by fit A in Table 1 and gives a χ^2/dof significantly less than one (any value of the strong coupling in the region $0.21 \leq \alpha_s \leq 0.25$ gives a comparable fit). Fits B and C give an idea of how much we can vary α_s and still get a reasonable fit.

Having obtained fits for a fixed α_s we proceeded to test whether the BFKL description could incorporate a running coupling. Though it is not clear how we should run α_s a plausible prescription is to run α_s with t. Fit D in Table 2 shows that we are not quite able to get an acceptable fit for running α_s. However, a reasonable fit (fit E) can be obtained if we fix the strong coupling appearing in the z variable defined in (11). This rather arbitrary choice corresponds to fixing the power which drives the dependence on the centre-of-mass energy (see (9)); it is a crude attempt to accommodate the proposal that higher order corrections may lead to a freezing of the power [8].

[1] GRV-94 LO [7].

[2] H1 has recently performed a similar fit where they found $\alpha_s = 0.22$ [4].

Fit	β	γ	$\chi^2_{J/\psi}$/dof	χ^2_{ϕ}/dof	χ^2_{ρ}/dof	χ^2/dof
D	217	17.1	1.1	2.8	0.5	1.4
E	460	0	0.8	0.8	1.1	0.9

Table 2: BFKL fits with a running coupling. Fit D runs the coupling with momentum transfer. In fit E, the coupling runs with momentum transfer in the α_s^4 prefactor whilst it is held fixed and equal to 0.14 in the definition of z.

The differential cross-sections for the three mesons are shown in Fig. 1, along with the preliminary ZEUS data [3]. The results for the LLA BFKL exchange thus suggest that the process $\gamma p \to VX$ is characterized by a strong coupling $0.17 < \alpha_s < 0.29$. We note that this is consistent with the value extracted from the Tevatron "gaps between jets" data [9].

Our results suggest that the largeness of $-t$ does allow a perturbative calculation to be performed however it is reasonable to ask whether it is necessary to invoke the full machinery of BFKL. Hence we also show the two-gluon exchange fits in Fig. 1. It is apparent that this approximation provides a very poor description of the light mesons, for which $\tau \sim 1$ over much of the range of data. Note that the model starts to give the correct quantitative behaviour away from $\tau = 1$. The two-gluon calculation only has one free parameter, α_s, and the differential cross-section has a fourth order power dependence on it. Note in particular that α_s has been fitted for each meson separately and also the unnatural trend that α_s falls as M_V falls, which means that running the coupling only makes the situation worse.

Using our best-fit values for the parameters given by Fit A in Table 1, we produced the predictions for the Υ and ω differential cross-sections shown in Fig. 2. We used fits B and C to estimate bounds on the possible values of the differential cross-sections. Data lying outside these bounds would indicate either a failure of the LLA BFKL formalism or a need to refine our treatment of the meson wavefunction.

The acceptable boundaries for the Υ are clearly much wider than for the ω. This is due to the fact that $M_\rho \sim M_\omega$ which means that $d\sigma_\omega/dt \sim (\Gamma^\omega_{ee}/\Gamma^\rho_{ee}) \cdot d\sigma_\rho/dt$. This scaling relationship between the ρ and ω is a model independent feature and also true of the two-gluon model. If other vector mesons, such as the ω, subsequently confirm our predictions it would suggest that in some instances even mesons made of light quarks can act as if they consist of two constituent quarks which share the meson's energy and momentum.

For all of the measured mesons, the quoted errors are about 10% of the absolute value of the differential cross-section. That we can fit the data suggests that the higher order corrections and the corrections due to the meson wavefunction should together contribute at most 10%.

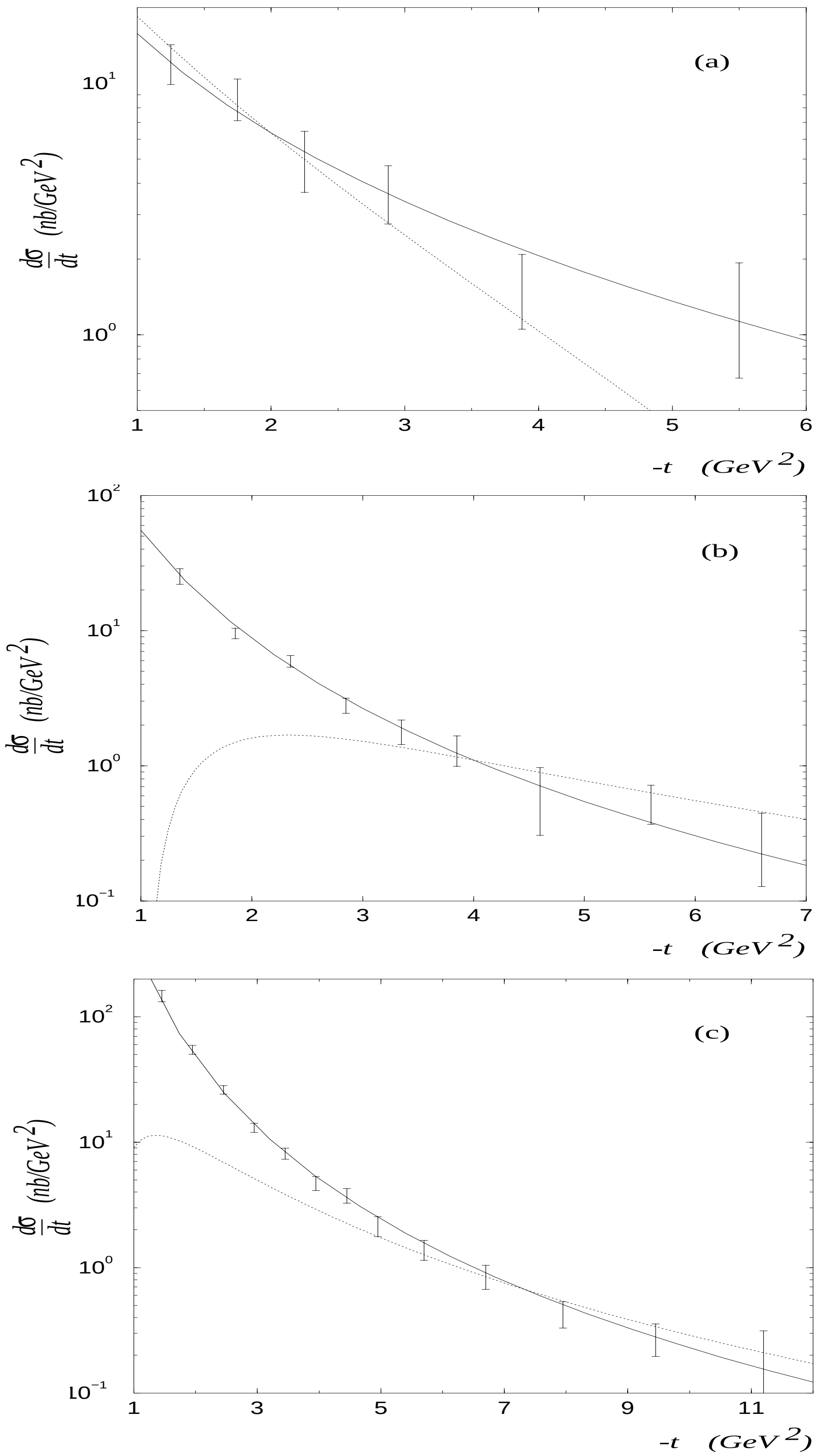

Figure 1: Comparison of BFKL (solid line) and two-gluon (dotted line) calculations compared to preliminary ZEUS data for (a) J/Ψ, (b) ϕ and (c) ρ meson production [3]. The BFKL curves correspond to Fit A, described in the text. The two-gluon curves are obtained by optimising α_s in each case, i.e. (a) $\alpha_s = 0.36$, (b) $\alpha_s = 0.35$, (c) $\alpha_s = 0.25$.

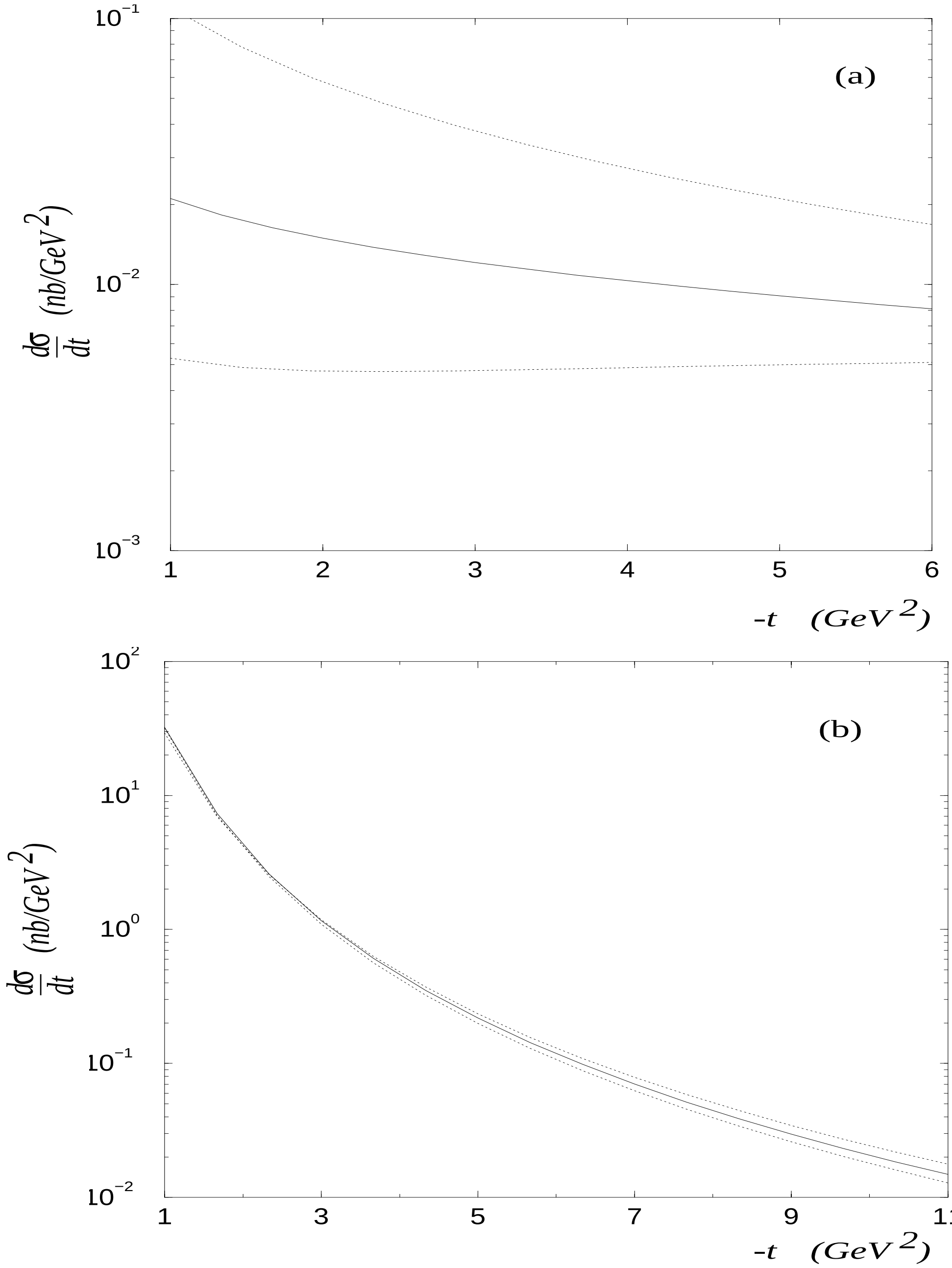

Figure 2: Predictions for (a) Υ and (b) ω meson production. The central lines correspond to BFKL Fit A, whilst the outer curves are obtained using Fits B and C.

Final Remarks

In [10] a LLA BFKL calculation incorporating a more sophisticated meson wavefunction than that used here is performed. It was concluded in [3] that this light meson calculation is incompatible with the available data. However, the author of [10] interprets the masses appearing in (6) as current masses (as ought to be appropriate for a truly perturbative calculation) which justifies the neglect of the quark mass for light mesons, rendering $|t|$ the only relevant scale. By interpreting the quark mass as a constituent mass, and assuming that the quark and antiquark share the meson momentum, we have demonstrated that the data can be understood provided the constituent mass is taken to be the scale in Λ. Another point of deviation is in the treatment of the strong coupling α_s. In [10] the coupling is a running coupling whilst we have considered a fixed coupling. The problem with running the coupling it is that we do not really know how it runs. Of course a complete description should incorporate a running coupling, but the work of [8] might be a hint that a fixed coupling may be appropriate for the BFKL exchange. The work of [10] has been developed in [11] where it is proposed that a large perturbative contribution arising from $q\bar{q}$ fluctuations in a chiral-odd spin configuration should play an important role in the region of the data. It remains to be seen if the simple model presented here can be justified within this approach.

Our results conclusively demonstrate that the two-gluon model is inadequate. The model predicts a dip at $\tau = 1$ which is not present in the data and this is the biggest obstacle to getting a decent fit. Corrections to the two-gluon approximation do fill in the dip (see [1] for more details on how this occurs) and further study is called for to establish whether a more sophisticated two-gluon calculation might work.

The model explored here has experimental tests to face in the future. If we can confirm our predictions for the Υ and ω it will be impressive. Unfortunately the outlook for data being obtained for the Υ in the immediate future is not good. The prospect for the measurement of the ω is better, since its cross-section is only a factor of ten down on the ρ. However, two pion decay which makes up $\sim 100\%$ of the ρ decays makes up about only $\sim 2\%$ of the ω decays. The result of this is that the ω is difficult to measure since it may require looking at the three pion decay of the ω in which it is harder to reconstruct $-t$. Data on the process $\gamma p \to \gamma X$, for which theoretical calculations have already been performed [12], should be obtained in the forseeable future. This process along with the ones considered here will continue to provide an important test of the validity of BFKL dynamics.

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